Robust inventory routing problem with variable travel times

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1 Introduction

Traditionally, supply chain activities such as supply management, inventory control, production and distribution are managed separately. However, the recent attempts to integrate and synchronize some of these activities resulted in a significant improvement of the competitiveness of a supply chain. A recent strategy in this context is the Vendor-Managed Inventory (VMI) system. In such a system the supplier is responsible for the inventory management of his customers. While the customers don't have to put effort into monitoring their inventory and passing orders, the supplier can combine multiple orders more effectively and thereby save inventory and distribution costs. As with most real-world systems, VMI system tends to be fraught with uncertainty. Uncertainty appears in the customer demands, the travel times, the manufacturing lead times, etc.

The central problem studied in this work is the multi-period Inventory-Routing Problem(IRP), a combined inventory management and routing problem, that supports VMI systems facing uncertain travel times. In a recent paper including robustness into an IRP, Aghezzaf (2008) [1] proposed to consider variability in both demands and travel times as stochastic stationary parameters. Solyali et al. (2012) [2] apply the robust approach, introduced by Bertsimas and Sim (2004) [3], to solve the IRP with stochastic demand. In our study, the uncertain travel times are assumed to be independent random variables that can take some values in a symmetrical interval around their nominal value. We propose four different approaches to tackle this problem. We further develop these approaches by adding heuristics to enhance their performance in terms of execution time and accuracy.

2 Modeling approach

2.1 Extended model

For the development of our models we start from a basic version of the IRP reviewed in Coelho et al. (2013) [4]. This basic model minimizes holding and transportation costs while providing a feasible distribution schedule. To avoid that the holding costs dominate the objective function, we extend the model with a penalty that has to be paid if the tours violate a certain deadline.

2.2 Solution methods

We propose four different models to face this slightly adapted version of the problem. In the first two models we identify the most interesting solutions in the search space. Subsequently we apply Monte Carlo simulation to determine the best solution. In the last two models we solve the problem through multiple iterations for an increasing set of scenarios. The comparison between solutions happens thus more directly.

In the first model we start from the optimal solution of the nominal problem. Subsequently we add constraints to the problem which enforce lower tours. The holding and transportation costs of the new solution will be higher than the ones of the solution of the nominal problem. However, the probability of paying a penalty is lower which can result in a lower overall cost. We can keep enforcing lower tours until the problem becomes infeasible. The result is a front of Pareto optimal solutions. On these solutions we apply Monte Carlo to find the best solution.

In the second model we use a similar strategy. The difference is the identification of the most interesting solutions happens not by enforcing lower tours but by adjusting the degree of robustness. For this purpose we reformulated the problem using the robustness approach developed by Bertsimas and Sim (2004) [3]. By starting from the nominal problem and then increasing the level of conservatism we obtain a similar front of solutions as in the previous approach. Subsequently we apply again Monte Carlo to obtain the best solution.

The third model is based on two stages. In the first stage the problem is solved using a set of scenarios. All these scenarios are represented by a portion of their cost in the objective function and for each scenario constraints are added to model their potential violation of the deadline. Based on the solution of the first stage new scenarios are found in the second stage and added to the scenario set. These new scenarios are typically among those for which the solution of the first stage does not perform well. Adding such scenarios ensures sufficient coverage of the range of variability. When enough scenarios are included in the scenario set, the solution of the first stage problem converges towards the optimal solution.

The fourth model solves the problem using a scenario set just like the third model. In contrary to the third model where new scenarios are searched in the second stage, the fourth model solves the problem for a set of random scenarios. This concept is also known as the sample average approximation (SAA) method [5].

To enhance the performance of our models we propose a number of improvements. For the first two models we present a proper stopping criterion. For the other two models we present several methods to cope with the increasing complexity of the problem. We design rules for the early fixation of variables, the removal of scenarios out of the scenario set, the use of antithetic variates to accelerate the convergence of the fourth model. . .

3 Results and conclusion

To validate the four models and compare their performance, we set up an experiment using a data set provided on www.leandro-coelho.com/instances/thesis/exact_irp. The main focus is on execution time and accuracy. For small instances the results indicate that the first model is the best both in terms of accuracy and execution time. For larger instances the first, second and fourth models are better. If accuracy is the most important criterion, then the fourth model outperforms the other ones.

Even with our heuristic improvements the results show an exponential increase in computational time when instances grow larger. Therefore, further research on improvements to speed up the methods would certainly be interesting. With this study we hope to stimulate other researchers to help developing these approaches further and to pursue the study of this fascinating problem.

Références

[1] E.-H. Aghezzaf. Robust distribution planning for supplier-managed inventory agreements when demand rates and travel times are stationary. *J Oper Res Soc*, 59(8) :1055-1065, 2008.

[2] O. Solyali, J.-F. Cordeau, G. Laporte. Robust inventory routing under demand uncertainty. *Transport Sci*, 46(3) :327-340, 2012.

[3] D. Bertsimas, M. Sim, The price of robustness. *Oper Res*, 52(1) :35-53, 2004.

[4] L.C. Coelho, J.-F. Cordeau, G. Laporte, Thirty years of inventory routing. *Transport Sci*, 48(1) :1-19, 2013.

[5] A. Shapiro, D. Dentcheva. Lectures on stochastic programming : modeling and theory (Vol. 9). SIAM, 2009.